EXPLORING LEARNERS' PRODUCTIVE DISPOSITION TOWARDS LEARNING MATHEMATICS: A PHENOMENOLOGICAL ANALYSIS

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ABSTRACT: *Mathematical proficiency is essential for students to navigate the ever-changing educational landscape effectively. This study searches into learners' productive disposition towards learning mathematics, addressing the research gap in understanding the emotional dimensions of mathematical proficiency. The research employs an interpretative phenomenological analysis (IPA), utilizing focus group discussions with twelve (12) learners, with six (6) from elementary and six (6) from secondary levels. Findings reveal five primary themes: emotional responses to learning mathematics, motivation and success, handling mistakes and overcoming challenges, attitudes towards collaborative learning, and the application of mathematics in real life and future aspirations. These themes shed light on learners' perspectives, providing insights into their experiences and challenges in the realm of mathematical proficiency. Understanding learners' productive disposition has profound implications for teaching and learning. Educators can tailor instructional approaches, incorporating collaborative learning, real-world applications, and strategies to foster motivation and resilience. This study contributes to the body of knowledge by illuminating the multifaceted nature of mathematical proficiency, paving the way for the development of targeted interventions to enhance learners' experiences in mathematics education.*

Keywords: mathematical proficiency, emotional dimensions, interpretative phenomenological analysis, focus group discussion, productive disposition, 4IR

I. INTRODUCTION

Mathematical proficiency, as defined by Kilpatrick, Swafford, & Findell [1], extends beyond the mastery of mathematical procedures; it encompasses a holistic and dynamic engagement with mathematical concepts in various contexts. Central to this proficiency is the concept of "Productive Disposition" — a term gaining prominence in educational research. Productive Disposition refers to learners' attitudes, beliefs, and emotional responses toward mathematics, influencing how they approach, engage with, and persist in mathematical tasks. Understanding and fostering Productive Disposition is vital not only for enhancing mathematical proficiency but also for preparing learners for the challenges and opportunities of the Fourth Industrial Revolution (4IR) [2].

In the context of the 4IR, characterized by the fusion of digital, biological, and physical technologies, the demand for individuals with strong mathematical skills is unprecedented. Mathematical proficiency, underpinned by a positive and productive disposition, becomes a cornerstone for success in a world where mathematical thinking is integral to problem-solving and innovation. Hence, exploring and comprehending learners' Productive Disposition is not only a pedagogical necessity but also an imperative for addressing the evolving needs of the 21st-century workforce [3].

The significance of Productive Disposition in developing Mathematical Proficiency lies in its ability to shape learners' approach to learning mathematics. Learners with a positive disposition are more likely to persevere in the face of challenges, view mistakes as opportunities for learning, and engage in collaborative problem-solving Woodward, Beswick & Oates [4]. This aligns with the broader goals of mathematics education, which aims not only to impart knowledge but also to cultivate lifelong learners capable of adapting to the demands of a rapidly changing world [5].

Numerous studies underscore the importance of considering learners' attitudes and emotions in the realm of mathematics. For instance, a study by Pekrun and Linnenbrink-Garcia [6] emphasized the significant impact of emotions on mathematical performance, demonstrating the interconnectedness of affective and cognitive domains. Despite the acknowledged importance of Productive Disposition, there exists a notable research gap in understanding its intricate dimensions and incorporating it into the development of mathematical proficiency assessment tools Philipp and Siegfried [7]. Many existing tools focus predominantly on cognitive aspects, overlooking the affective and emotional components that significantly influence learning outcomes Wei, Saab & Admiraal [8]. This study aims to address this gap by conducting an in-depth exploration of learners' Productive Disposition through the lens of Interpretative Phenomenological Analysis (IPA). This research gap is not merely theoretical; it has practical implications for educators and curriculum designers. Without a nuanced understanding of learners' emotional responses, motivations, and attitudes toward mathematics, the effectiveness of teaching strategies and interventions may be compromised Joyce and Cartwright [9]. Thus, this study seeks to contribute to the field by providing insights that can inform the development of more holistic and targeted mathematical proficiency assessment tools. Several related studies emphasize the importance of investigating Productive Disposition using IPA, highlighting the need for a qualitative approach that captures the lived experiences of learners Love, Vetere & Davis [10]. For instance, Lei, Cui & Zhou [11] employed IPA to explore students' attitudes toward mathematics, revealing rich narratives that quantitative

methods might overlook. The employment of this methodology is justified by the depth and nuance it brings to the understanding of learners' experiences, aligning with the complexity of Productive Disposition.

II. METHODS

Research Design

This study employs Interpretative Phenomenological Analysis (IPA) as the research design. IPA is particularly appropriate for exploring the lived experiences and perceptions of individuals within a specific context, making it well-suited for our investigation into learners' Productive Disposition towards learning mathematics, Alase [12]. IPA focuses on the detailed examination of participants' subjective experiences, providing a rich and in-depth understanding of the phenomenon under investigation, Pietkiewicz and Smith [13]. Given the intricate and multifaceted nature of Productive Disposition, IPA enables a nuanced exploration of the emotional responses, motivations, and attitudes of learners toward mathematics.

Research Participants

The participants for this study consist of twelve (12) learners, purposefully selected to provide a comprehensive understanding of Productive Disposition across different educational levels. Six (6) participants are drawn from elementary schools, specifically Opol Central School, and the remaining six (6) from secondary education, specifically Opol National Secondary Technical School. The criteria for participant selection include enrollment in the school year 2022-2023, active involvement in mathematics subjects, and attendance at a large public school in the Philippines, ensuring diversity in experiences and backgrounds.

Data and Collection

The use of focus group discussions (FGDs) as the data collection method is apt for this research, given its exploratory nature. FGDs provide a platform for participants to express their experiences, perceptions, and emotions collectively, encouraging rich, dynamic, and interactive discussions [14].

In this study, FGDs were conducted separately for the elementary and secondary level learners to create a comfortable environment for participants to freely share their insights and experiences. Prior to the FGDs, a well-structured FGD questionnaire (Table 1) was developed, and validated by three (3) mathematics experts, ensuring the content's alignment with the research objectives. The FGD questionnaire was framed to revolve around the theme of "How learners perceived learning mathematics," which aligns with the productive disposition strand of mathematical proficiency. The conduct of FGDs began with a comprehensive orientation process, which included seeking permission from both the school administration and parents, particularly since the respondents were minors in age. Informed consent was obtained, emphasizing the voluntary nature of participation. Proper ethical considerations were meticulously followed to ensure the privacy and confidentiality of the participants, and no personal information was disclosed in the analysis and reporting of results.

The FGDs were carried out over two days, allocating separate sessions for the elementary and secondary level learners. Day 1 involved the collection of signed ascent forms and the orientation proper, lasting for an hour for each group. The orientation aimed to familiarize participants with the FGD process, emphasizing the importance of honest and open sharing while maintaining a respectful and inclusive environment. Day 2 marked the actual FGD sessions, with each group participating for an hour. The sessions were recorded using voice recorders, with participants' consent, ensuring that no participant was identifiable in the audio recordings. The recordings were later transcribed, translated from the vernacular to English, and meticulously reviewed.

Table 1. FGD Questionnaire

Research Questions
1. How do you feel about learning mathematics?
2. How do you feel when your teacher gives you a mathematics problem to solve?
3. Can you describe a situation when you felt successful in your mathematics class?
4. How do you feel about making mistakes in mathematics class?
5. Can you describe a situation when you felt challenged in your mathematics class?
6. Can you describe a situation when you had to work collaboratively with others to solve a mathematics problem?
7. What are your thoughts about the mathematics tasks or activities your teacher gives you?
8. Can you describe a situation when you had to apply mathematics skills outside of the mathematics classroom?
9. How do you think your mathematics skills will help you in your future life and career?

Data Analysis

Thematic analysis is a systematic approach used to identify and report patterns (themes) within qualitative data, providing an insightful understanding of the phenomenon under investigation, Braun and Clarke [15]. This method was adopted in this study to analyze the voice-recorded FGD sessions. The thematic analysis process involved several steps, ensuring rigor and reliability. First, the recordings were transcribed verbatim. For participants who spoke in the local language, the transcriptions were then translated into English to maintain consistency across the analysis process. Following this, the researchers employed open coding, initially generating codes by examining the transcripts line by line. Then, similar codes were grouped to create preliminary themes. The next steps in the analysis process included refining and defining the themes, ensuring that they were coherent, consistent, and captured the essence of the data. This process was guided by an iterative approach, with researchers returning to the data repeatedly to gain deeper insights and confirm the validity of the themes, Braun and Clarke [15]. The researchers named and defined the emerging themes using the translated transcript and the coded data. Subsequently, they compiled the data analysis based on the collected data and these evolving themes. The participants were coded for anonymity and identification purposes, using labels such as L1 for Learner 1, L2 for Learner 2, and so

forth. This coding system allowed for a systematic organization of the data, facilitating the analysis and presentation of the findings.

III. RESULTS AND DISCUSSION

Emotional Responses to Learning Mathematics

The first emergent theme revolves around the learners' sentiments and challenges in learning mathematics. The participants expressed a range of emotions and experiences related to their engagement with mathematical concepts. Several participants conveyed negative emotions associated with learning mathematics. For instance, L1 expressed, "*I don't like it, the symbols and formulas.*" echoing common apprehension tied to mathematical symbols and formulas Keziah, Salinas, Tolibao and Moneva [16]. This sentiment reflects the anxiety and discomfort some students feel with the symbolic language of mathematics. Similar sentiments were echoed by L4, who found math "confusing" and specified challenges with fractions and decimals, areas frequently cited as challenging [17].

Conversely, some participants expressed positive emotions. L2 shared, "*I feel very excited learning math, especially if the teacher introduces a new lesson.*" This aligns with the idea that positive experiences and engaging teaching methods contribute to students' enthusiasm for mathematics Kilpatrick et al., [1]. L8 mentioned feeling "progressive and on the road to success," linking a positive attitude to a sense of progress and achievement, Haerens [18] Success experiences, as highlighted by L8, are known to boost confidence and foster a positive attitude toward mathematics, Chen, Bae, Battista, Qin, Chen, Evans & Menon [19]. The findings emphasize the diverse ways students engage with mathematics, highlighting the need for educators to recognize and address these emotions. Negative emotions can hinder learning, [20], suggesting the importance of implementing strategies to alleviate math-related stress. Positive emotions are conducive to a conducive learning environment [21]. Understanding what triggers positive responses can guide educators in creating more effective and enjoyable mathematics instruction, aligning with existing literature on affective factors in mathematics learning [22].

Motivation and Success

The second theme centers on the motivation of students in the context of mathematics learning and the perceived connection between motivation and success. The responses provide valuable insights into the factors influencing students' motivation and their perceptions of success in the domain of mathematics. Participants identified various factors influencing their motivation in mathematics. L3 emphasized the importance of real-world applications, stating, "*If I see how math is used in real life, I am motivated to learn more*." This aligns with research indicating that connecting mathematical concepts to practical situations enhances motivation and understanding, Abramovich, Grinshpan, & Milligan [23]. L7 expressed motivation through the challenge, stating, "*I like the challenge. When it's hard, I want to solve it.*" resonates with achievement goal theory, Chazan, Pelletier & Daniels [24]. Regarding success, participants' definitions varied. L5 equated success with understanding, stating, "*I feel successful when I understand the lesson completely*." This aligns with research highlighting the significance of conceptual understanding Kilpatrick et al. [1]. In contrast, L9 associated success with high grades, stating, "*I feel successful when I get good grades. That's how I know I'm doing well.*" Recognizing diverse definitions of success, including conceptual understanding and achievement outcomes, Elliot and Dweck [25], is crucial. Educators must acknowledge and address these diverse perspectives, fostering a learning environment that accommodates various indicators of success, Sanger [26]. The findings align with literature emphasizing the role of motivation in mathematics learning, Moddleton [27]. Motivation is not a one-size-fits-all concept, and educators should consider individual differences when designing interventions.

Challenges and Overcoming Difficulties

The third salient theme centers around the challenges students face in mathematics learning and the strategies they employ to overcome these difficulties. Participants highlighted struggles with abstract concepts, with L2 stating, "*Sometimes, the abstract stuff is confusing. It's like a puzzle without the picture*." echoing existing research, Borghi, Binkofski, Castelfranchi, Cimatti, Scorolli & Tummolini [28]. L6 expressed frustration with time constraints, stating, "*Tests are timed, and that stresses me out. I can do it, but I need more time.*" Time-related stress aligns with studies indicating that timed assessments can create anxiety and hinder performance [29].

Participants shared strategies for overcoming difficulties. L1 emphasized seeking help, stating, "*I ask the teacher or friends when I don't get it. Getting different explanations helps.*" aligning with the importance of social interaction and collaborative learning in mathematics, Schwarz, Swidan, Prusak & Palatnik [30]. L8 advocated for breaking down complex problems, stating, "*When it's too hard, I break it into smaller steps. It's easier to solve step by step*." supporting problem-solving strategies suggested in literature, Ortiz [31] and the idea of metacognition, Chiu, Jones, & Jones [32]. Understanding challenges is crucial for educators. Addressing the struggle with abstract concepts may involve instructional approaches using visual aids and practical, Stylianides and Stylianides [33]. Time-related stress highlights concern about the negative impact of timed assessments on student well-being, prompting the exploration of alternative assessment methods. Overcoming difficulties emphasizes the importance of a supportive learning environment and aligning with established pedagogical practices that enhance students' ability to navigate challenges in mathematics [34].

Attitudes Towards Collaborative Learning

The fourth theme elucidates students' attitudes and experiences regarding collaborative learning in the context of mathematics education. Participants provided insights into

the dynamics of group work, highlighting both positive and challenging aspects. L5 shared a positive sentiment, stating, "*Working in a group can be helpful. Different perspectives make it easier to see things differently*." aligning with research emphasizing the benefits of diverse perspectives in problem-solving, Slavin [35]. L1 echoed this positivity, expressing, "*I like discussing problems with others. It helps me gain new insights*." highlighting how collaborative learning enhances problem-solving skills and promotes a sense of community, Medina, Buan, Mendoza & Liwanag [36]. Challenges in collaborative learning were also voiced. L3 noted, "*Sometimes, group work is frustrating. Not everyone pulls their weight*." emphasizing the importance of addressing group dynamics to ensure equitable participation, Gamit, Antolin & Gabriel [37]. L7 shared a related concern, stating, "*I feel anxious when I don't understand, and the group is moving fast*." highlighting the need for scaffolding and support within group settings, particularly when students have varied levels of understanding, Janneke van de Pol and Volman, [38]. Understanding students' attitudes toward collaborative learning is crucial for effective instructional design. Positive experiences emphasize collaboration as a means to enhance problem-solving skills and foster shared learning responsibility, Reyes [39]. To address challenges, educators should intentionally structure group activities, provide clear guidelines, promote inclusive participation, and support students feeling overwhelmed, van Diggele, Burgess & Mellis [40]. Effectively structured collaborative learning can be a powerful tool in mathematics education, optimizing environments for mutual learning and the development of interpersonal and problem-solving skills.

Application of Mathematics in Real Life and Future Aspirations

The fifth theme delves into the participants' perspectives on the practical application of mathematics in real-life scenarios and its influence on their future aspirations. Understanding the perceived relevance of mathematics to everyday life is crucial for educators to enhance engagement and demonstrate the broader significance of mathematical concepts. Several participants expressed a positive outlook on the practical relevance of mathematics. L2 stated, "*Math is everywhere. It helps us solve real-world problems*." aligning with research emphasizing the importance of illustrating the applicability of mathematical concepts Go [41]. However, contrasting views were also present. L8 expressed skepticism, saying, "*I don't see how algebra will help me in real life*." emphasizing the need for interventions that broaden students' awareness of the diverse applications of mathematics, Yao and Manouchehri [42]. Participants provided insights into how their perceptions of mathematics influenced their future aspirations. L4 shared, "*I want to pursue a career in engineering because I love applying math to solve problems*." highlighting the impact of positive experiences with mathematics on career choices, [43]. In contrast, L6 mentioned, "*I avoid careers that involve a lot of math. It's just not for me*." indicating the need for interventions that broaden students' awareness of the diverse applications of mathematics in various fields, Liu [44]. Understanding and addressing students' perceptions of the application of mathematics in real life can play a crucial role in fostering a positive and purposeful learning experience, ultimately influencing their attitudes toward the subject and shaping their future aspirations.

IV. CONCLUSION AND RECOMMENDATION

In conclusion, this study explores learners' productive disposition toward learning mathematics through an interpretative phenomenological analysis. The findings underscore the intricate interplay of emotions, motivations, challenges, collaborative learning dynamics, and the application of mathematics. Recommendations for educators include addressing emotional responses, tailoring motivation and success strategies, overcoming challenges, optimizing collaborative learning, and bridging the gap in application. This contributes to ongoing discourse on mathematics education, providing actionable insights for educators to enhance the quality of mathematics instruction and foster a positive and purpose-driven learning experience for students. Future research should continue exploring the evolving landscape of learners' productive disposition toward mathematics, considering dynamic educational contexts and emerging pedagogical trends.

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